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SIMIODE Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations

TEACHER VERSION
SNAILS IN A TIDE POOL

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Abstract: This lab asks students to use linear differential equations to model temperature change, both in a sand tide pool and inside the shell of a snail in that tide pool. We offer data on temperature in a tide pool as the sun heats the water and ask students to model the tide pool's water temperature with constant rate of change. The solution to the water temperature model is then used to model the internal temperature of a snail caught in the tide pool. Students are asked to produce and solve a linear first-order, non-homogeneous, ordinary, differential equation for the temperature inside a snail's shell using Newton's Law of Cooling (or heating here). This project takes students through the entire modeling process, from building a model to analyzing and then creating alternate water temperature models. This classroom modeling scenario can be expanded to have students collect their own water temperature data in a lab setting or to further explore the alternate models.

Keywords: linear growth, Newton's law of cooling/warming, biology, modeling

Tags: linear, first-order, non-homogeneous, temperature, snails, data collection

STATEMENT

After the tide rises and then falls, tide pools that occasionally trap mollusks and other organisms are left in the sand or rocks. Because the volume of the tide pool is small, the sun heats the tide pool at a much faster rate than it heats the ocean. Mollusks are ectothermic organisms, which means their body temperature matches the temperature of their environment. Thus, the temperature of the mollusk starts at the initial water temperature of the ocean. After the tide recedes and leaves a tide pool and snail behind, the snail warms along with the water in the tide pool. Typically snails remain in a tide pool for approximately 3 to 4 hours without any new water being added to cool

t (minutes)	T (degrees Celsius)
0	28.7
7	31.1
12	31.9
19	32.7
32	34.2
37	34.7
43	34.9
53	35.4
60	35.8

Table 1. Tide pool water temperatures, T, as recoded from a simulated tide pool.

the pool. The purpose of this project is to use differential equations to model a snail's temperature change and to determine whether it reaches its upper lethal body temperature while in the tide pool.

Part I. Modeling Water Temperature in a Tide Pool

Option A. Building a Model from a Table of Data

The data in Table 1 includes water temperature recorded from a simulated tide pool with mud snails. Our goal is to create a model for how the water temperature T changes with time. The water may not change at a perfectly constant rate, but one model we want to use is one in which $\frac{dT}{dt}$ is constant.

1. Use Excel or a graphing calculator to plot the data.
2. Assuming the water temperature changes at a constant rate w, give an appropriate value for w based on the data in Table 1. Justify your value in your lab report.
3. Use the value w you found, along with the initial value given in the table, to solve the initial value problem below.
$$\frac{dT}{dt} = w, \quad T(0) = T_0$$
where $T(t)$ is the temperature of the tide pool water in degrees Celsius at time t in minutes.
4. Plot your solution $T = T(t)$ with the data and include this plot along with the function $T = T(t)$ in your lab report.

Is the equation you gave for T a reasonable model for water temperature based on the data? What other models might work? Write ideas here with your group and include a summary of this discussion in your lab report.

t (minutes)	T (degrees Celsius)

Table 2. Blank table to be filled in with tide pool water temperatures.

Option B. Collecting Water Temperature Data and Building a Model

We will simulate the sun heating a tide pool in our lab. Our goal is to create a model for how the water temperature T changes with time. The water may not change at a perfectly constant rate, but one model we want to use is one in which $\frac{dT}{dt}$ is constant. After creating your simulated tide pool, move it to your heat source. Record the temperature of the water every 3-5 minutes for approximately 20-30 minutes. Fill in the empty table below with times and water temperatures as you collect this data. Note that for a simulation of mid to southern Atlantic coast water in the summer months, initial water temperature should fall in the range of 75-85 degrees Fahrenheit or 23.9-29.4 degrees Celsius [1].

1. Use Excel or a graphing calculator to plot the data.

2. Assuming the water temperature changes at a constant rate w, give an appropriate value for w based on the data you recorded in Table 2. Justify your value in your lab report.

3. Use the value w you found, along with the initial value given in the table, to solve the initial value problem below.
$$\frac{dT}{dt} = w, \quad T(0) = T_0$$
where $T(t)$ is the temperature of the tide pool water in degrees Celsius at time t in minutes.

4. Plot your solution $T = T(t)$ with the data and include this plot along with the function $T = T(t)$ in your lab report.

Is the equation you gave for T a reasonable model for water temperature based on the data? What other models might work? Write ideas here with your group and include a summary of this discussion in your lab report.

Part II. Modeling the Body Temperature of a Snail

According to [2], the upper lethal body temperature of an amphibious snail is between 40 and 45 degrees Celsius. NOAA weather data shows that typical water temperatures in the summer months for the mid-Atlantic coast range from 23.9 to 29.4 degrees Celsius [1]. For this part, we will assume the snail's body temperature matches the surrounding water temperature in the tide pool and use $S = S(t)$ to denote the internal temperature of the snail in degrees Celsius at time t in minutes.

1. Create a model for the snail's temperature, $S(t)$, using information from Part I and include in your lab report.

2. Find the amount of time it will take for the snail's temperature to reach the range of the upper lethal body temperature according to your model and include this in your lab report.

3. Using the same rate w from Part I, find the maximum initial water temperature T_0 that will allow a snail to survive in a 4-hour tide pool. (Pay attention to units.) Does the maximum allowable T_0 fall into the range of typical water temperatures for the mid-Atlantic coast? Discuss the implications of your findings in your lab report.

Do you think it is realistic to assume the snail's body temperature would be the same as the water temperature? Why or why not? Write a section of your lab report that answers these questions.

Part III. Modeling Snail Temperature with Shell Insulation

Most mollusks have shells for protection which may also provide some insulation from the heat of the water. As a result, their internal temperature may change at a different rate than the water. In 2015, biology and mathematics faculty at the SIMIODE East Developer's workshop (held at Virginia Wesleyan College in Norfolk, VA on the mid-Atlantic coast) collected snails and measured their body temperatures along with water temperature in a simulated tide pool. They found that during each temperature reading (after the initial reading), the temperature of the snail was cooler than the surrounding water. According to Newton's Law of Cooling (or heating up here!), the rate of change of the temperature of the snail is proportional to the difference between the temperature of the water and the temperature of the snail. Here we will create a new model for $\frac{dS}{dt}$ which accounts for the insulating factor of the shell.

Option A:

1. Create a differential equation for the snail's temperature $S = S(t)$ using $\frac{dS}{dt}$ and Newton's Law of Cooling. Use the the water temperature model, T, from Part I and constant of

proportionality k.

2. Solve your (linear) differential equation, recalling that the initial temperature of the snail may be assumed to be the same as the initial temperature of the water.

3. During the snail experiment, it was determined that after 51 minutes the temperature of the snail was 33.6 degrees Celsius. Use this information to determine k. This should involve solving an algebraic equation that you can't do by hand; you may use WolframAlpha.com or a computer algebra system for assistance in determining a value for k.

4. Graph $S(t)$ along with $T(t)$. How does the snail's temperature compare to the water temperature? Is the shell providing significant insulation? How long can the mollusk remain in the tide pool before it reaches its upper lethal body temperature? Does the snail reach its upper lethal body temperature before new cool water arrives at the tide pool?

5. Review the assumptions you used to build the model for $\frac{dS}{dt}$. Do you still believe these assumptions are realistic?

In your report, give your model for $\frac{dS}{dt}$ with both its analytic and graphical solutions. Write paragraphs that answer all questions. Specifically, be sure to describe how the temperature of the snail changes in comparison to the temperature of the water and to discuss the validity of your model so far.

Option B:

1. Create a differential equation for the snail's temperature $S = S(t)$ using $\frac{dS}{dt}$ and Newton's Law of Cooling. Use the the water temperature model, T, from Part I and constant of proportionality k.

2. Solve your (linear) differential equation, recalling that the initial temperature of the snail may be assumed to be the same as the initial temperature of the water.

3. Additional data is required in order to estimate the constant k. Instead, we will investigate the internal snail temperature using various values of k. Graph $T(t)$ along with $S(t)$ using each of the following values for the parameter k: -1, -0.5, -0.1, 0.01, 0.05, 0.1, 0.2, 1. Based on the graphs of S, which values of k can you rule out and which values of k seem reasonable? Explain your reasoning.

4. Experimental data suggests that k is somewhere between 0.05 and 0.5. Select a value of k in this range, graph $S(t)$ along with $T(t)$. How does the snail's temperature compare to the water temperature? Is the shell providing significant insulation? How long can the mollusk remain in the tide pool before it reaches its upper lethal body temperature? Does the snail reach its upper lethal body temperature before new cool water arrives at the tide pool?

5. Review the assumptions you used to build the model for $\frac{dS}{dt}$. Do you still believe these assumptions are realistic?

In your report, give your value for k and your model for $\frac{dS}{dt}$ with both its analytic and graphical solutions. Write paragraphs that answer all questions. Specifically, be sure to describe how the temperature of the snail changes in comparison to the temperature of the water and to discuss the validity of your model so far.

Part IV. Analysis of Models and Further Questions

In Part III we examined the validity of our model and here we explore alternative models.

1. Discuss whether or not it is realistic to assume that water temperature increases at a constant rate for the entire duration of the tide pool. Consider all factors that might affect water temperature.

2. Create a non-constant model for water temperature in a tide pool during the afternoon before the next tide based on the data in Part I. How does changing your temperature model affect the differential equation for $\frac{dS}{dt}$? Is it still a linear differential equation? (You do not need to solve for S using the new model.)

3. Suppose the water temperature in the tide pool levels off after a certain time. What conditions on $T(t)$ would be needed for the snail to survive in this case? Sketch by hand a graph of $T(t)$ and $S(t)$ in this scenario, keeping in mind Newton's Law of Cooling.

Write a section of your lab report that includes the answers to these questions

REFERENCES

[1] National Oceanic and Atmospheric Administration. 2016. *Water Temperature Table of All Coastal Regions*. Available at `https://www.nodc.noaa.gov/dsdt/cwtg/all_meanT.html`. Accessed August 2016.

[2] Burky, Albert J., Pacheco, and Eugenia Pereyra. 1972. Temperature, Water, and Respiratory Regimes of a an amphibious snail *Pomacea Urceus (Muller)* from the Venezuelan Savannah. *Biological Bulletin*. 143: 304-316.

COMMENTS

This activity has the potential for in class laboratory activities, out of class investigations, and even an in depth research project for biology and mathematics students. Two student versions are available, one that follows *Option A* throughout and another for *Option B*. Student Version A, which follows *Option A*, uses previously collected data and asks students to build and analyze a model for snail temperature. This data was collected by the authors with the assistance of Dr. Deirdre Gonsalves-Jackson, a marine biologist at Virginia Wesleyan College. Snails were collected using the Virginia Wesleyan College research vessel and analyzed on the college campus. Ocean water was used in a simulated shallow sand tide pool that was placed in direct sunlight on a hot July afternoon in 2015 and water and snail temperature data was recorded for 2 hours. With this data and linear growth model, the snail does not survive in a tide pool for 3-4 hours. (Our snails did survive the two hour experiment!) It's possible that the simulated tide pool from which the data was collected had too high of a rate of change of the water temperature, or more likely, that water temperature leveled off after a certain time period and the increasing temperature model cannot be used for the full 3-4 hours. At the end of the handout students are asked to discuss the validity of the assumptions and the model, since we are pretty sure that snails are not dying off at alarming rates in tide pools. This option takes students through the entire modeling process, causing them to revisit assumptions about their model and investigate the outcomes of those assumptions. We feel this is a very valuable experience for the students, who are used to building models with predictable outcomes.

In Student Version B, which follows *Option B*, students collect their own water temperature data in a lab setting and build a model with those parameters. This scenario has more variability in potential outcomes, possibly allowing snails to survive. Students also experiment with various values of k, the parameter modeling the shell's insulation capabilities, and select a value based on their observations rather than on recorded snail temperature data. The authors would be very interested to hear the values of water temperature obtained in further trials of this part of the lab. This lab could also be extended to a research project between biology and mathematics students who can recreate a tide pool with snails and measure both water and snail temperatures.

Part I. For *Option B*, to create a simulated tide pool students will need sand and ocean water which can be created using Instant Ocean salt. When VWC students completed this lab, they coated the bottom and sides of a clear container with sand, filled it with the simulated sea water, and used a heat lamp to raise the temperature.

A third option which reduces the time required for the in class activity is to provide the students with a value for w or an equation for $T(t)$ rather than using or collecting data. A suitable range for the rate w for the data from *Option A* is 0.10 to 0.12.

Part II. With the model $S(t) = wt + T_0$ and the data from *Option A*, the snail reaches a lethal body temperature in 94 to 113 minutes depending on the value of $w \in [0.10, 0.12]$ chosen. Explorations of adjusting the starting temperature of the water to simulate different times of year or various regions of the coast could be included here.

Part III. Next, assuming the shell provides insulation and using Newton's Law of Cooling, the students should obtain

$$\frac{dS}{dt} = k(T - S) = k(wt + T_0 - S)$$

with solution

$$S(t) = ce^{-kt} + wt + T_0 - \frac{w}{k} = \frac{w}{k}e^{-kt} + wt + T_0 - \frac{w}{k}$$

where $S(0) = T_0$ determines $c = w/k$. Here dS/dt is a linear differential equation that requires integrating factors and integration by parts to solve. Alternately, the linear differential equation could be solved with Laplace transforms. At each stage, students must remember constants of integration. If they miss the constant in integrating dT/dt for example the final $S(t)$ will show a preliminary drop in temperature which should allow students to catch their error. *Option A* in Part III has students solve for k which requires Wolfram Alpha or a computer algebra system while *Option B* asks students to look at the graphs and investigate $S(t)$ for various values of k. The approach in *Option B* for choosing an appropriate value for k could also be used in *Option A* if desired.

For completeness, we include Table 3 with additional temperature data collected for a second snail. However, we caution the use of this data as it was collected from a snail in a different simulated tide pool and thus the water temperature was different from that recorded in Table 1.

t (minutes)	S (degrees Celsius)
0	28.7
30	32.6
47	34.7
49	34.9
83	36.7

Table 3. Temperature, S, inside a snail's shell as recoded from a simulated tide pool.

Part IV. Alternate water temperature models that students might try include polynomial models or a piecewise function that is linear and then constant. Depending on the models being considered, this project could be revisited to find new models for dS/dt and explore their solutions in more depth.

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